THE PERFORMANCE OF HYDRAULIC SOFTWARE SOLUTIONS THROUGH SUDDEN VELOCITY CHANGES

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ABSTRACT

The Auckland Council 2011 Stormwater Flood Modelling Specifications recommends modelling software based on the St Venant equations, but warns "Performance of hydraulic structures should always be cross-checked with manual calculations or other software". The New Zealand Standard NZS 4404:2010 goes further and states "Stormwater systems shall be designed by calculating or computer modelling backwater profiles" using the Bernoulli solution. Verification Method E1/VM1 published by the Department for Building and Housing also specifies the Bernoulli solution. It seems hydraulic modellers are faced with two incompatible standards, but on closer examination the incompatibility is resolved if "manual calculations or other software" is taken to involve the Bernoulli equation. Further, the Auckland specifications associate "energy losses due to turbulence" with "any sudden changes in velocity (magnitude and direction) e.g. at transitions, junctions, bends, entrances, exits, and obstructions." Since the St Venant equation has nothing to do with energy, this expands recommended use of the Bernoulli equation beyond engineered structures to natural rapid changes in cross-section, such as at lake and swale exits and channel overfalls. Examples are discussed of errors associated with the misuse of the St Venant equations in such cases.

KEYWORDS

Standards, St Venant, Bernoulli, discontinuous flow solutions

PRESENTER PROFILE

Alastair Barnett has over forty years of experience applying hydraulic modelling in twenty countries. His award-winning *AULOS* design software has been proven through the last twenty years of analysis of linked surface and subsurface network problems such as hydropower surge dynamics and urban drainage failures, including tsunami inundation.

1 INTRODUCTION

In hydraulics there is a classic history (Rouse & Ince, 1963) of confusion between the momentum principle and the energy principle. This applies particularly at points of sudden velocity change, where all the flow properties pass through a region of rapid variation – see the photographic examples in Appendix A.

Henderson (1966) advised "The energy equation always holds true, provided proper allowance is made for the energy "losses" (more properly, the dissipation of kinetic energy into heat energy), in writing it down. Similarly the momentum equation always holds true, provided proper allowance is made for all forces acting." Barnett (1994) went further, showing analytically that (subject to an exact mass balance) a balance of energy was a necessary condition for a balance of momentum and vice versa.

1.1 SUDDEN FLOW VARIATION TYPE 1: IMPOSED BY BED GEOMETRY

In most of the examples in Appendix A, the sudden velocity change is forced by strong variations in the channel cross-section, as channels are not in general prismatic in character. Photograph 1 illustrates the bed and wall irregularities typical of a natural channel.

Photograph 1: Irregular natural channel through a coral reef, Kiribati

Flow through such channels is continually being modified by bed and wall forces, arising from both normal and shear stresses. To account accurately for all such forces, as required in a momentum analysis, is an almost impossible task even in a study reach as limited as that in the photograph.

However, being associated with a fixed bed, these forces all contribute zero work, so can be eliminated from a work-energy analysis. The task of setting up an energy balance then reduces to consideration of the reduction in total energy head from point to point along the channel, and it is a comparatively simple job to develop a combination of friction losses and expansion or contraction losses which can easily be calibrated for a serviceable predictive model.

1.2 SUDDEN FLOW VARIATION TYPE 2: IMPOSED BY FLOW MISMATCH

This device of eliminating all bed and wall forces is the key to successful analysis of all of the cases shown in Appendix A, except for that illustrated by Photograph A11. Here a sudden velocity change is apparent as a hydraulic jump some distance upstream of the obvious geometrical constriction imposed on the cross-section by the overbridge pier.

Such jumps have been extensively studied in the laboratory and at full scale. In New Zealand, the need to predict the behavior of shock fronts in hydropower canal systems led to a major field investigation involving some 100 man-years of resources.

For example, Photograph 2 illustrates a travelling jump formed after a simulated dambreak involving the sudden opening of a control gate (background, centre) in a major irrigation canal.

Photograph 2: Artificial Dambreak Surge, Rangitata Diversion Race

Here the sudden change in velocity appears in a channel of fairly uniform cross-section, with the most obvious visible attribute a sudden change in water level, giving a jump approaching 1m in height. In this particular case the bed and wall forces are predominantly shear, and these can be conventionally accounted for by resistance terms. The net normal forces can be attributed to the change in depth, so the momentum equation is quite tractable, as proved by close matching between field results and momentum-based solutions.

This situation is the basis of the St Venant (1871) analysis, which was formulated to study tidal surges in the estuarine reaches of the River Seine. As is made clear in the definitive textbooks (e.g. Cunge et al., 1980) it is a mistake to extend the historical treatment of this special (Type 2) case to Type 1 sudden flow variations instead of using Bernoulli analysis as prescribed in more modern standards.

For example, Barnett (2003) showed that St Venant analysis successfully predicted the movement of dambreak wave fronts as measured in the laboratory, but *only* if Bernoulli analysis was used for treatment of outflows through the breach aperture.

2 TREATMENT OF SUDDEN CHANGES AS DISCONTINUITIES

Where the length of the region of rapid variation is small (for example, the transition region at the outlet of the culverts in Photograph A5), it is convenient to treat the rapid variation as a discontinuity in a macroscopic view of the whole flow system. From this viewpoint, an energy "loss" is seen as a step decrease in the total energy head of the flow, applied at the point of step cross-section change between culvert pipe and open channel.

However, in many cases the length of the transition region may be significant, as with the hydraulic jump shown in Photograph 2, where the initial jump is followed by a trail of lesser undulations before regular flow is restored. The pool-to-channel constriction shown in Photograph A1 also applies over an appreciable length of channel, although changes will be more significant under flood flows than in the low flow conditions pictured.

Here analysis can still be simplified to a step change, so that a balance can be struck between regular flow conditions upstream and downstream of the transition provided the details of flow within the transition have little influence.

Unfortunately this does not apply to the use of the St Venant equations for a pool-to-channel constriction, as shown by laboratory studies carried out by the University of Canterbury (Barnett et al., 2004).

Photograph 3: Laboratory pool-to-channel transition

Laboratory flows were measured to within 1% accuracy, and use of the simplified Bernoulli solution (as specified in Verification Method E1/VM1) gave flow results about 1% below the laboratory values. Use of the full Bernoulli solution gave results about 1% above laboratory values, requiring correction by a small contraction loss coefficient K=0.03, well within book range (Henderson, 1966).

2.1 FALSE PUMP EFFECT

In contrast the *AULOS* St Venant solution for flow was 17% high when treating the 2m long transition as a discontinuity. This overestimate of channel capacity is described as a "False Pump" effect, as an unphysical step energy *gain* is added to the downstream flow purely by the action of the computational solution. Energy cannot be gained by a water flow except when a real pump is operating, and strictly even this does not supply an energy gain, but an energy transfer from external electrical or chemical energy sources.

If the 2m transition is split into two 1m reaches, the flow error reduces to 10%, while to provide a solution comparable to the Bernoulli solution, the St Venant solution requires computational steps reduced to 0.1m. While this is possible in this case, although barely practical, similar grid refinement of actual cross-section discontinuities (such as in Photograph A6) is hardly feasible.

3 DEMONSTRATION PROBLEM

The false pump propensity of the St Venant solution means that conduits designed using this solution will be literally substandard, as they will be smaller than those designed using the Bernoulli solution as required by Verification Method E1/VM1 and by NZS4404:2010. Increasing sole reliance on the St Venant solution threatens systemic underdesign of much of the New Zealand storm water system.

The significance of the flow error for any modelling package can be assessed by using it to set up and run a demonstration problem (Barnett 2012).

3.1 THE PROBLEM

An existing open drain is rectangular in cross-section and 9m wide. Between 10m and 110m from the coast the drain runs through higher ground, so a narrower channel reach has been cut through, based on a 1.8m diameter semicircular bottom shape with vertical walls above the open semicircle. Downstream of the cutting, the channel invert is at datum level, and upstream of the cutting the channel invert is at 0.100m above datum, giving a constant bed slope of 0.1% to the connecting semicircular channel reach. See Figure 1.

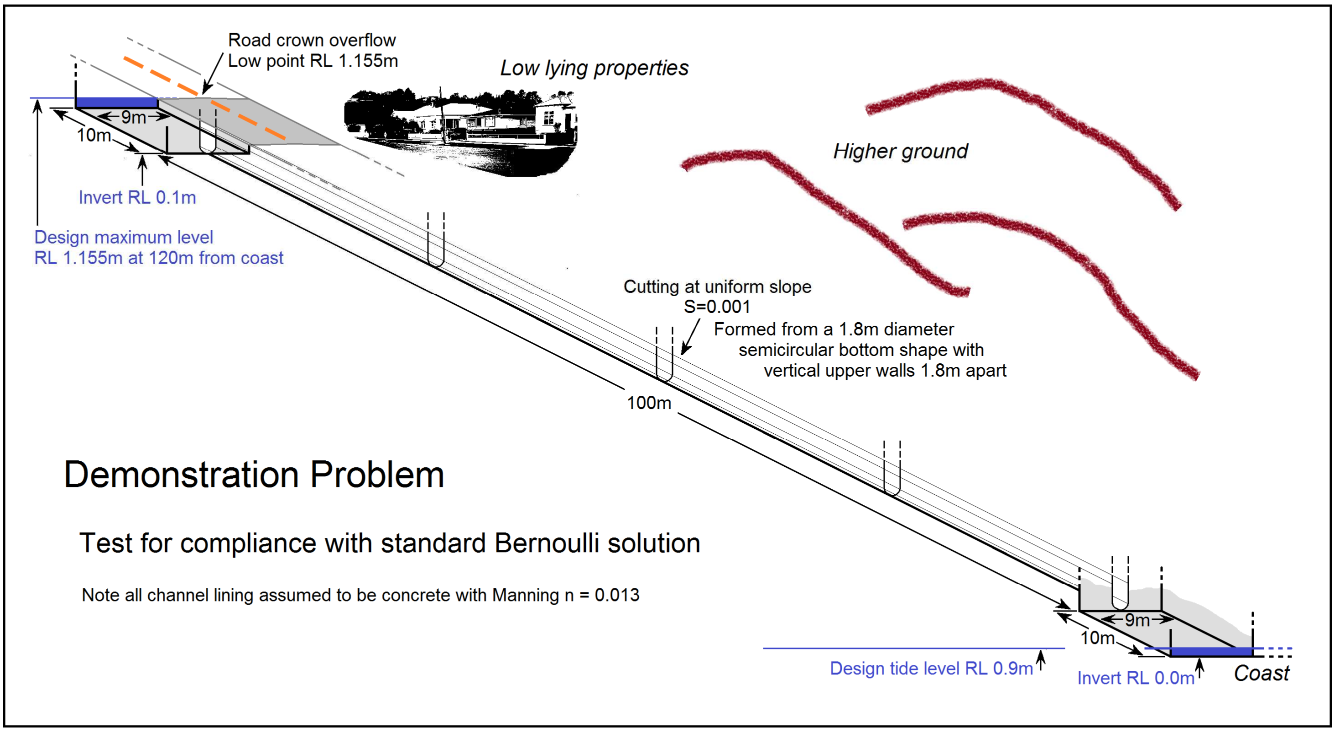

Figure 1: Layout of Demonstration Problem

The channel lining is concrete throughout, with a Manning n of 0.013. The semicircular channel reach is square ended in plan at both ends.

At the coast the static water level applying for flood design calculations has been assessed as 0.9m above datum, and upstream of the higher ground a road runs beside the drain, with a minimum road crown level of 1.155m above datum measured at 120m from the coast. The Council is concerned about possible flooding of low-lying properties behind the road, and has commissioned a study to recalculate the flood flows for various return periods.

They now require a flow capacity check of the last 120m of the drain so they can determine if the revised 100 year ARI flood flow will overtop the road crown.

3.2 SOLUTIONS

Application of the simplified Bernoulli solution in Verification Method E1/VM1 gives a standard drain flood capacity of 1.817 cumecs. If this was the 100 year flood peak, then it follows that according to the standard the road crown levels will provide marginal protection against a flood of this ARI. The *AULOS* full Bernoulli solution computes virtually the same answer.

Any higher computed capacity must be provided by the introduction of additional energy to force the extra flow through the channel by False Pump effects. A computed capacity of around 2.5 cumecs is found by the *AULOS* St Venant solution, meaning that if this is the required standard capacity for a 100 year flood, the drain will have a substandard actual capacity for only a 20 year ARI flood. Expanding the St Venant equation into a 2D Shallow Water Equation analysis makes matters even worse: if a computed 100 year capacity of around 3.0 cumecs is required for the 100 year flood, the drain will have a substandard actual capacity for only about a 10 year flood.

Full details can be found in the original submission in Barnett (2012), copies of which have been widely circulated around the modelling community.

If expressed in terms of levels, a flow of 2.5 cumecs will obviously overflow the road using Bernoulli analysis, and this overflow will reduce the upstream levels according to the longitudinal profile of the road crown. If the road is sandbagged to prevent overflow, the predicted level at 120m is 16 cm above the low point, which in practical terms means one layer of sandbags will barely hold back the flooding. For the flow of 3 cumecs estimated by the 2D equations, the level at 120m is about 26 cm above the low point, which will require two layers of sandbags to contain the flooding.

If sandbags cannot be deployed in time, assuming the overflows correspond with a dip in the road levels of 0.1m with a parabolic shape over 20m gives overflows of 0.35 cumecs for the 2.5 cumec channel flow and 0.64 cumecs for the almost 3 cumec channel flow. This corresponds with 50% and 58% respectively of the excess inflows, with the balance escaping to the coast driven by the increased upstream levels.

4 STORMWATER SYSTEM MODELLING OF AUCKLAND CBD

It is encouraging to see that coupled surface/subsurface flow modelling has at last been resumed in the Auckland CBD. The original coupled model was developed using Mike 11 UD by the then Auckland City Development Consultancy – see Naidu and Kearney (1995). A substantial programme of remedial works resulted from the study, including the re-routing of storm water drainage around the new underground Britomart rail terminal. Another coupled model in St Heliers was documented by Essex and Joynes (1995).

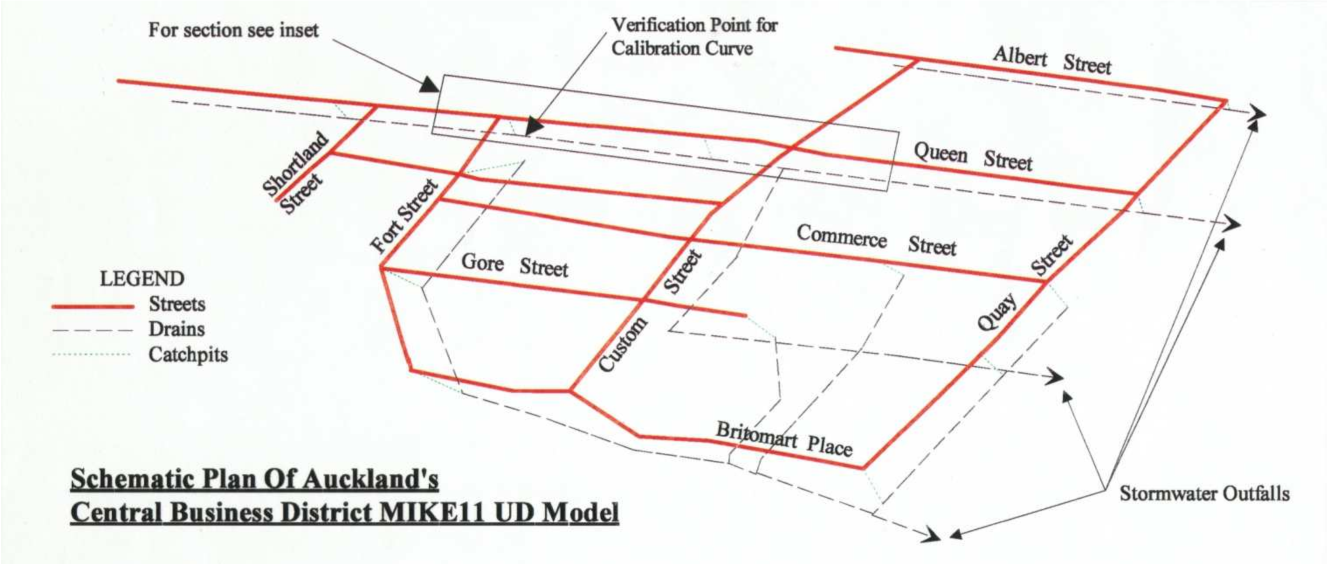

Figure 2: Schematic of coupled model of Auckland CBD

Figure 2 shows the coverage of the CBD model, which was based on the flat reclaimed part of the city in an area with known stormwater flooding problems. Reference is made to a section along Queen St, and this is shown in Figure 3.

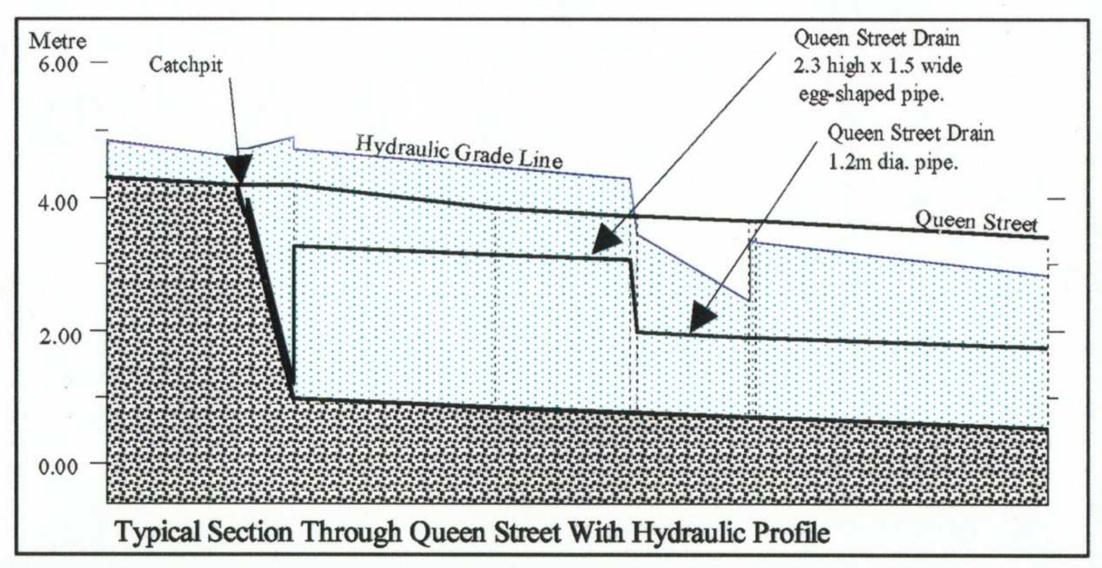

Figure 3: Longitudinal section showing an area of predicted flooding

The old egg-shaped pipe is clearly a bottleneck downstream of the Fort St catchpit, so that the catchpit is discharging flow back into Queen street instead of draining the street.

Where the Queen St drain changes to a smaller circular pipe, the flow goes through a constriction, increasing the velocity head and hence lowering the water level. The hydraulic gradient is significantly steeper, but a lower downstream head is available because of the bifurcation of the Queen St drain into the Custom St drain and lower Queen St drain. This bifurcation diverts significant flow along Custom St, so the velocity head is then reduced through the lower Queen St drain.

This analysis all depends on Bernoulli – what could be expected from St Venant?

These Bernoulli model predictions are backed by calibration. Figure 2 also identifies a verification point for a calibration curve, and this is shown in Figure 4.

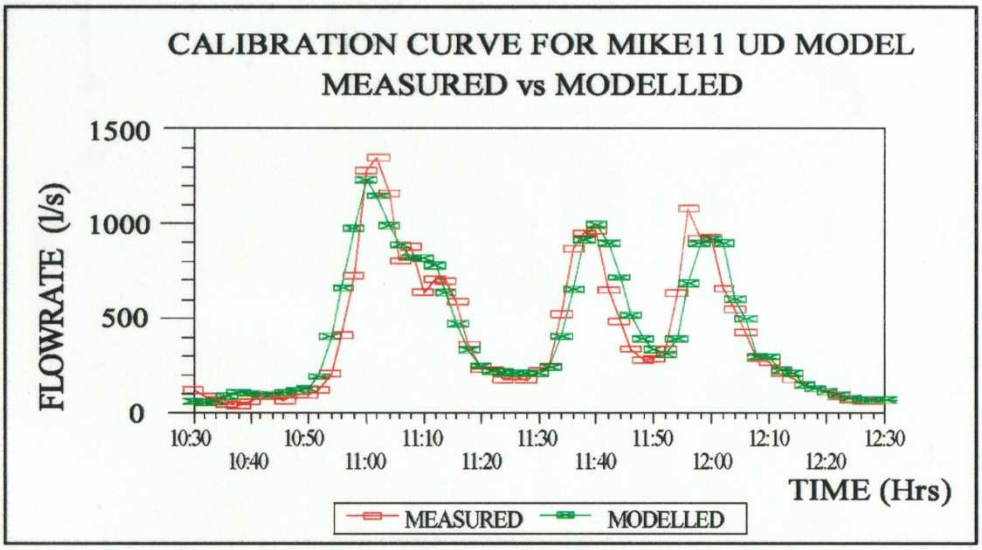

Figure 4. Example of model calibration

This calibration below the junction of Queen St and Fort St used flow metering from a multi-peaked storm on 27 April 1993. Bearing in mind flows in this area are influenced by tides as well as catchment outflows, the calibration is excellent. We trust that comparable calibrations can be obtained for the new models, but the above analysis suggests that will be difficult to achieve if those models rely principally on St Venant analysis.

5 CONCLUSIONS

1. Bernoulli and St Venant analysis are often confused, but the two types of analysis differ significantly in the treatment of sudden flow variations.

2. St Venant analysis was formulated in 1871 to study tidal surges up the river Seine, but it is a mistake to use this for general design in place of more modern New Zealand standards, which specify Bernoulli analysis for stormwater drainage design.

3. Laboratory studies demonstrate that St Venant solutions overestimate channel conveyance capacity through channel cross-section transitions such as pool outlets. Energy analysis clearly shows unphysical energy gains in such solutions, which require input of energy from a false pump.

4. The significance of such flow modelling errors can be assessed for any software package by trial on a given demonstration problem. This produces significant flow and level errors if proper Bernoulli analysis is not applied.

5. The original coupled surface/subsurface Auckland models complied with NZ verification standards, but compliance of the new coupled models awaits proper demonstration.

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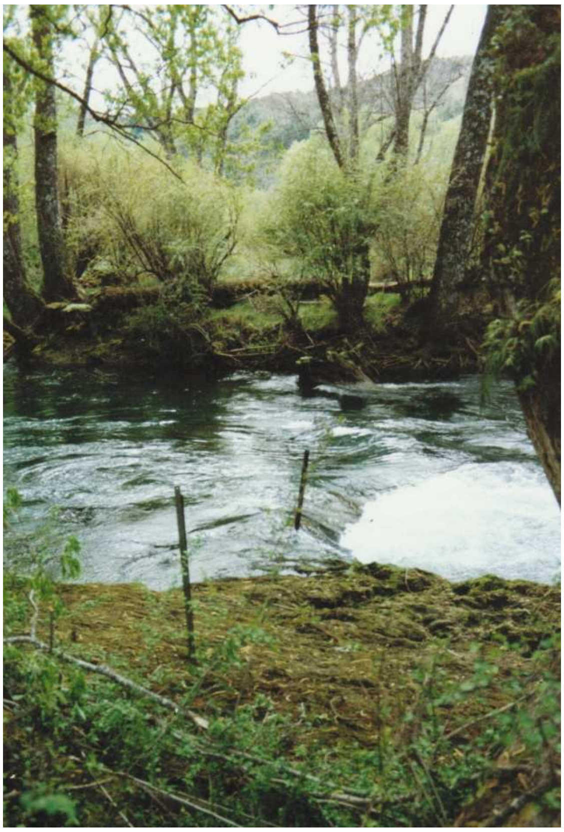
APPENDIX A: EXAMPLES OF SUDDEN VELOCITY CHANGES


Photograph A1: Pool to channel


Photograph A2: Pool to pipe culvert


Photograph A3. Power station to channel


Photograph A4: Pool to overfall


Photograph A5: Pipe(s) to pool


Photograph A6: Pool to V-Notch

Photograph A7: Road drain to small culvert

Photograph A9: Road Drain to large culvert

Photograph A8: Small culvert to road drain

Photograph A10: Overbank flow from river

Photograph A11: Channel under bridge