OVERCOMING SPATIAL-TEMPORAL RAINFALL VARIATION USING RAINFALL RADAR

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ABSTRACT

It is nearly impossible to accurately quantify rainfall variability across a stormwater or sewer catchment using discrete point rainfall measurements. The variability across the catchment can be significant depending on the catchment location and surrounding terrain and the nature of the meteorological processes producing the rain. For many hydrological applications, such as sewer inflow and infiltration modelling, extrapolation and interpolation of point rainfall measurements is standard practice and is one of largest unknowns in the model. Decisions about the techniques used for extrapolation, as well as the adequacy of the conclusions drawn from the modelling results, depend heavily on the magnitude and the nature of the uncertainty involved.

In this paper we will outline our recent investigation using accurate short range radar in an attempt to quantify how standard point rainfall measurement and extrapolation techniques effect sewer model calibration and eventually options resulting from the model. In the highlighted case study we completed a detailed sewer model calibration using current industry best practice. As a second work stream we obtained radar data from the University of Auckland's short range mobile radar unit for the entire monitoring period. We then tested the model calibration using the "true" rainfall distribution over each sewer-catchment as identified from the radar and commented on the variation in model calibration parameters and how the different rainfall distribution effects the perceived system performance and potential options analysis.

KEYWORDS

Rainfall Radar, Hydraulic Modelling, Sewer infiltration, Wastewater Flow Prediction

1 INTRODUCTION

It is common across New Zealand and other parts of the world to build hydrologic and hydraulic models of sewer and storm water infrastructure for planning purposes. The models are typically used to develop comprehensive master plans across the catchment which drive large capital expenditures for mitigating current system performance issues (e.g. combined sewer overflows, flooding) and planning for future growth. Having a robust calibrated hydrologic and hydraulic model is critical to planning cost effective and focused solutions.

Accurate rainfall accumulations are therefore an essential boundary condition for all these models. Too often rainfall data quality is taken for granted and the spatial variability in the data is often not well understood. In the case of New Zealand storms generally show high spatial and temporal variability which is difficult to capture using typical discrete point rainfall measurements. The traditional engineering approach to obtaining rainfall boundary conditions is to make use of tipping bucket rain gauges at a density of approximately 1 every 2 to 4 km^2 depending upon the catchment terrain. However, this standard is often not adhered to due to cost and over large regions this is not always practical.

An additional concern with using rain gauges is that a collection of sparse point measurements may not be able to properly characterise the extreme spatial gradients which are known to exist in precipitation fields (Morrissey et al., 1995, Steiner, 1996, Nystuen, 1998, Villarini et al., 2008).

Essentially, hydrologically significant rainfall may either "fit between" rain gauges, in which case it is not sampled, or it may be incident on members of a gauge network but not present in unmeasured areas, in which case oversampling occurs. Either scenario will bias the rainfall boundary conditions in sewer models, leading to poorer model predictive skill.

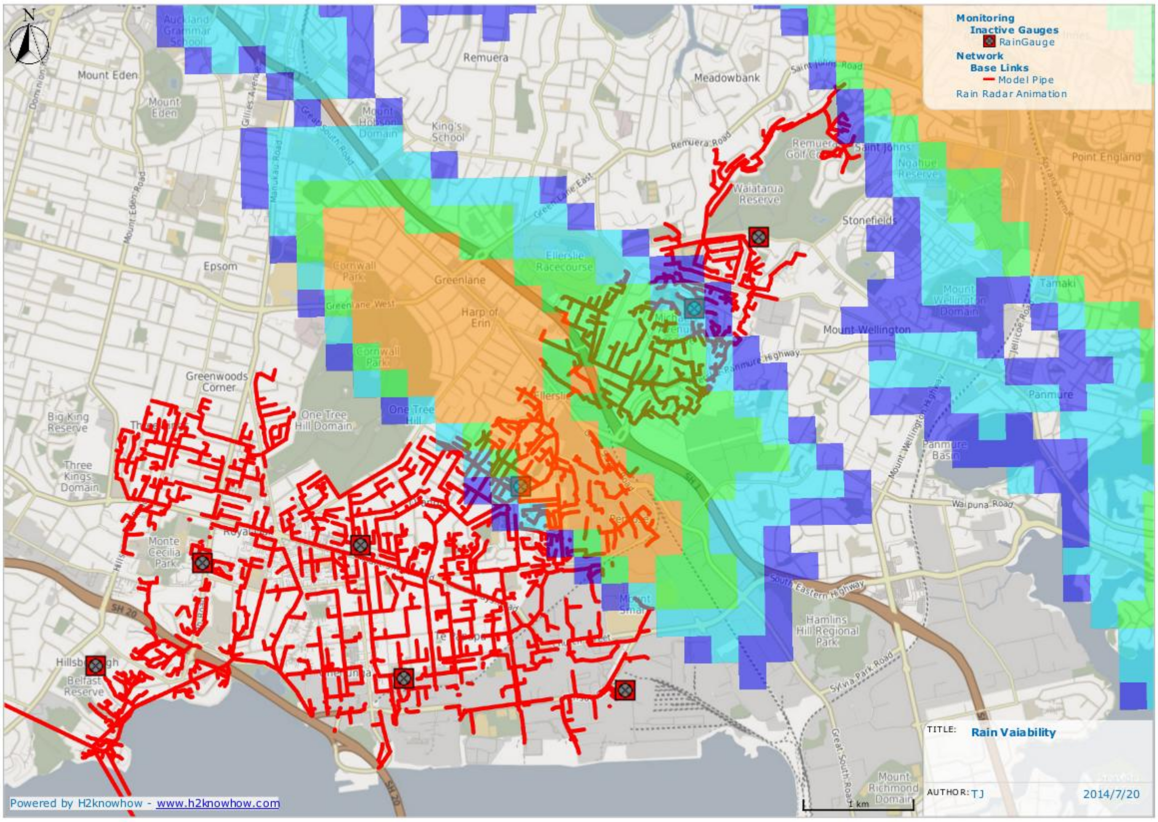

Figure 1:Rainfall Variability Example (12/09/2013 19:08)

A complimentary source of rainfall information is available from weather radar measurements. Weather radar generates spatial maps of rain location and infer instantaneous rainfall rate by measuring the intensity of reflection (backscatter) of electromagnetic radiation off falling raindrops (after Marshal 1953 and Marshal and Palmer, 1948). Careful processing of radar data is necessary to retrieve of surface rainfall rate from radar reflectivity measurements made aloft (for a recent review, see Villarini and Krajewski, 2010). Some of these sources of error, such as, the uncertainty in the observed rainfall's drop size distribution (Twomey, 1953, Battan, 1973, Atlas et al., 1999), beam blocking (Harrold et al., 1974, Andrieu et al., 1997) and uncertainty in the knowledge of the vertical distribution of rain (Fabry et al., 1992, Kitchen et al., 1994, Joss and Lee, 1995) and the resulting effects on the estimation of rainfall rate have been researched extensively over the past decades. Comparison of radar retrieved estimates of rainfall with point rain gauge measurements can indicate how well these errors have been accounted for and corrected.

For most engineering applications the best estimates of surface rainfall accumulation depth can be made by combining both radar and rain-gauge measurements (Wright et al, 2014). These composite fields contain both point rain rate information from direct in-situ rain gauge measurements and information about the spatial distribution of rainfall from radar measurements and can be prepared in raster formats suitable for ingestion into distributed models.

International work has highlighted the modelling improvements made possible by these composite measurements. Lowe et al (2014) reported improvements in urban runoff modelling when using composite radar-gauge fields over the same rain gauge only measurements. The improvement in spatial sampling afforded by radar measurements can offset radar uncertainties and result in improvements in model response. Sempere-Tores et al (1999) compared radar only and rain gauge only data for driving combined sewer system (CSS) flow models and found that radar data better reproduced observed flow, despite some point wise disagreements with rain gauge measurements. The extra spatial information contained in radar measurements of rainfall has also been put to use modelling pollution buildup and runoff (Shaw et al 2010) and forecasting sewer overflow risk (Heinonen et al 2013).

In this work we investigate the impact of high resolution rain radar and rain-gauge fields on a network sewer model of the Onehunga catchment in Auckland New Zealand. In the analysis we utilized a traditionally calibrated hydrologic/hydraulic model to make an assessment on how discrete rainfall measurements might skew calibration parameters when spatial rainfall variation is persistent.

2 STUDY AREA

The study area is located on the south east of the Auckland Isthmus. The total contributing area is approximately 2,107 ha (from Project Storm 2) and accommodates a total population of 46,776 (2006). Approximately half of the catchment is residential, while industrial and open space covers nearly 20% each and the remaining 5% area is commercial activities. The catchment terrain is relatively flat with some minor elevation gains in the north east portion of the catchment.

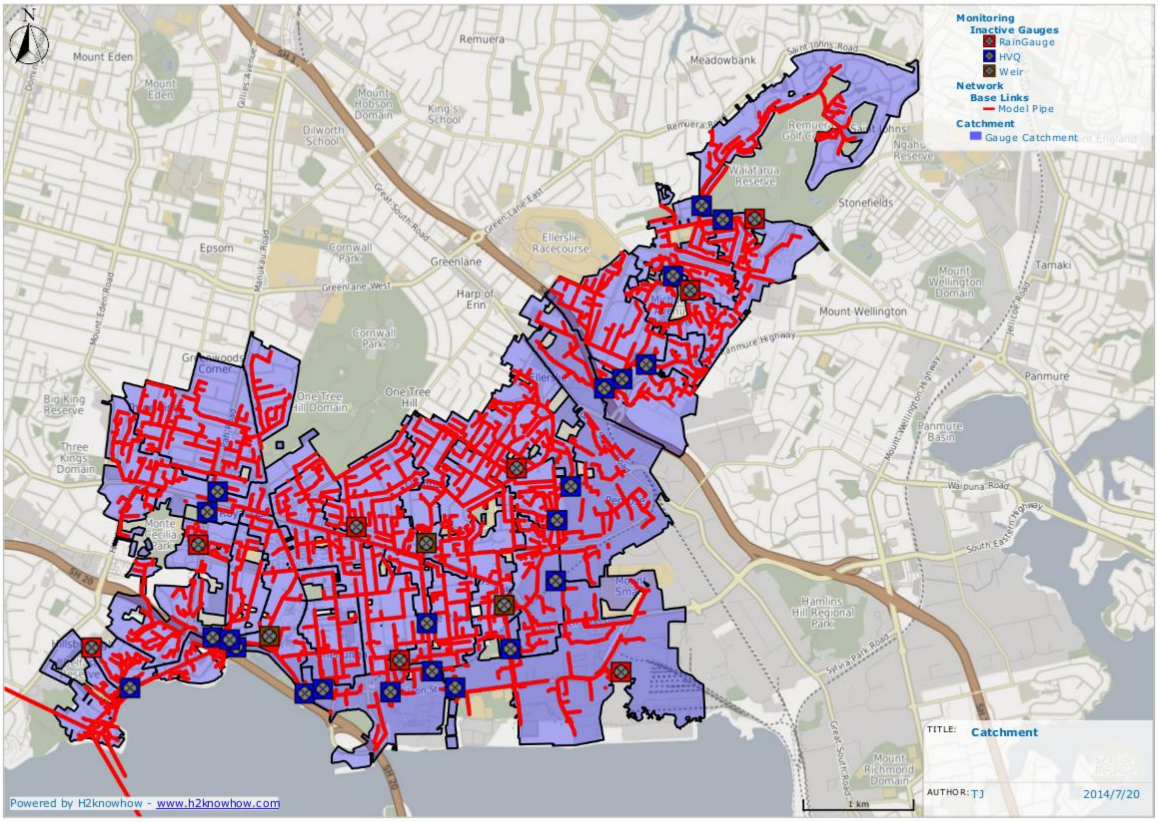

Figure 2: Catchment Boundary, sewer network (red), flow monitoring locations (blue squares) and rain gauges (red squares)

3 METHODOLOGY

3.1.1 RADAR DATA COLLECTION

For this analysis radar observations were collected with the University of Auckland High Resolution "Trailer Radar". The "Trailer Radar" consists of a fully articulated 1.8m diameter radar dish mounted on a short tower, coupled by flexible waveguide to a 25kW masthead transceiver, the outputs of which are in turn fed, along with information regarding the dish direction, into a PC housed in a small operator's cab. The radar system is entirely self-contained on a tandem axle trailer. The total mass is about 2.5 tonnes, allowing it to be towed by a light four wheel drive vehicle; provided that the trailer's hydraulic breaking system is used. The radar mast is folded down onto the trailer for transport. A complete description of the radar system and discussion of its suitability for small catchment monitoring may be found in Sutherland-Stacey et. al. (2011).

For this project the Trailer Radar was deployed to the harbour outfall in the Mangere WWTP (Figure 3). The site affords an uninterrupted field of view over the upper reaches of the Manukau harbour to the study catchment. The study catchment's orientation relative to the field site is such that the catchment's major axis (12km) coincides with the down-range direction.

Figure 3:The University of Auckland Trailer Radar Setup at Mangere WWTP

The radar was configured to obtain a radar scan of the catchment every 30 seconds during rain events. Raw radar observations spanning 3 months (2013/08/29 to 2013/10/25) was collected. The main source of error in the radar estimates of rainfall is the uncertainty in radar calibration and variation in rainfall in the vertical. The radar dish angle was set to 6 degrees so the radar beam climbs from about 200m to 2000m elevation over the length of the catchment so the height at which rainfall is sampled varies over the catchment introducing a range dependent bias which depends on the weather type (depth of rain).

The radar estimates of rainfall accumulation were processed onto a grid with 250 m X 250 m pixels and 2 minute intervals. Rain events were automatically detected by grouping periods of continuous rainfall and the range dependent bias was then estimated per rain event by comparison between rain gauge measurements and their corresponding radar pixels. Per-event bias correction addresses variations in rain type between different weather systems- for example large scale condensation has a very different vertical distribution to convectively driven rain storms.

3.1.2 RADAR DATA POST-PROCESSING

Following the automatic correction (calibration) process, the bias corrected radar accumulations were output into a standard ERSI ASCII raster format for each time 2 min time interval. All of the rasters were loaded into the master geospatial database and standard database tools were used query and aggregate each raster into 5 minute rainfall accumulation time series for each grid cell that intersected the catchment. In all 337 (2,107 Ha / 250 m^2) unique time series were generated. This allowed the full utilisation of the radar data in its most un-aggregated form.

3.1.3 HYDRAULIC MODEL

In this analysis a previously calibrated Innovyze InfoWorks CS model of the catchment was used to compare traditional rainfall measurements and radar measurements. The model is considered a detailed catchment model and contains nearly all of the available pipes in the network and has been calibrated using industry best practice. The calibration data was collected over the winter of 2013 with 25 flow monitoring points and 8 rainfall gauge locations.

The time series generated from the radar was directly imported into the hydraulic model. Each time series was assigned to its intersecting sub-catchment in the model (2,337 sub-catchments in total). Each sub catchment was only assigned a single grid cell from the radar raster with no splitting. As the average catchment size in the model is 0.75 ha vs the 6.2 ha radar grid cell size a single cell to catchment matching technique was considered appropriate. The radar data was supplemented by gauge rainfall data for the seeding period in the model. This provided the antecedent conditions in the model and assured that the model was producing the realistic flows from each catchment prior to the introduction of the radar rainfall.

The rainfall-runoff transformation for each sub-catchment is composed of two separate and unrelated hydrological processes: a fast response model and slow response model. Each model (fast and slow response) is split into a volume model and a routing model. The volume model is used to complete a general mass balance between total rainfall and losses (e.g. initial losses, evaporation, and infiltration) and the routing model transforms the excess rainfall into a runoff rate. The two volume models used for the fast response component was the Fixed PR model (for impervious surfaces) and the New UK model (for pervious surfaces) and the large catchment model was used for routing for fast response. The ground water infiltration model was utilised for the slow response component. Characteristic lag times between rainfall and a local flow model response are minuets and hours to days for the fast and slow models respectively. The runoff generated from the hydrological model at each sub-catchment is directed to a single manhole and is routed through the network using the Saint Venant equations. More information on the models can be found in the InfoWorks user manual.

During the model calibration the rainfall recorded at the nearest rain gauge (8 in total) was used to general runoff from each of the 2,337 catchments. The rainfall from radar was introduced into the model as a collection of rain gauges located at the centre of each intersecting radar grid cell which comprised of 337 pseudo-rain gauges.

4 RESULTS

For this analysis we have chosen to utilise a rainfall event on 12th September 2013. This event was chosen as a good representative event with good variance between the rainfall gauges readings. On average 25 mm rain fell over the catchment between 6:00 and 14:00 on the 12th which is considered an average winter event for the catchment. The event was not used in the rain-gauge calibration or validation process which also made it attractive for the analysis.

4.1.1 RADAR/RAIN COMPARISON

Prior to running the model a comparison between the rainfall measurements from radar and rain gauge was performed. Figure 4 below shows an example radar calibration plot for a rain gauge in the catchment. The accumulated rainfall at the grid cell corresponding to the location of the physical rain gauge vs the accumulation seen at the gauge is shown in plot below. As seen in the Figure 4 a good calibration was achieved at gauge ONERG06. Table 1 below shows the radar calibration for the rainfall event on 04/09 which will be used to further this analysis. As seen in Table 1 not all of the rainfall gauges showed a calibration match as good as ONERG06. This is a result of several issues but most likely that the resolution of the rainfall radar (6.2 Ha) may be difficult to be directly transfer to a discrete point rainfall measurement. It is also noted that the rainfall gauge location within the radar cell was not examined in detail. The rainfall gauge could be at located at the extremity of the associated cell and may have a better match with an adjacent cell. In general the bias correction ensured that the average difference across all rain gauges was minimised.

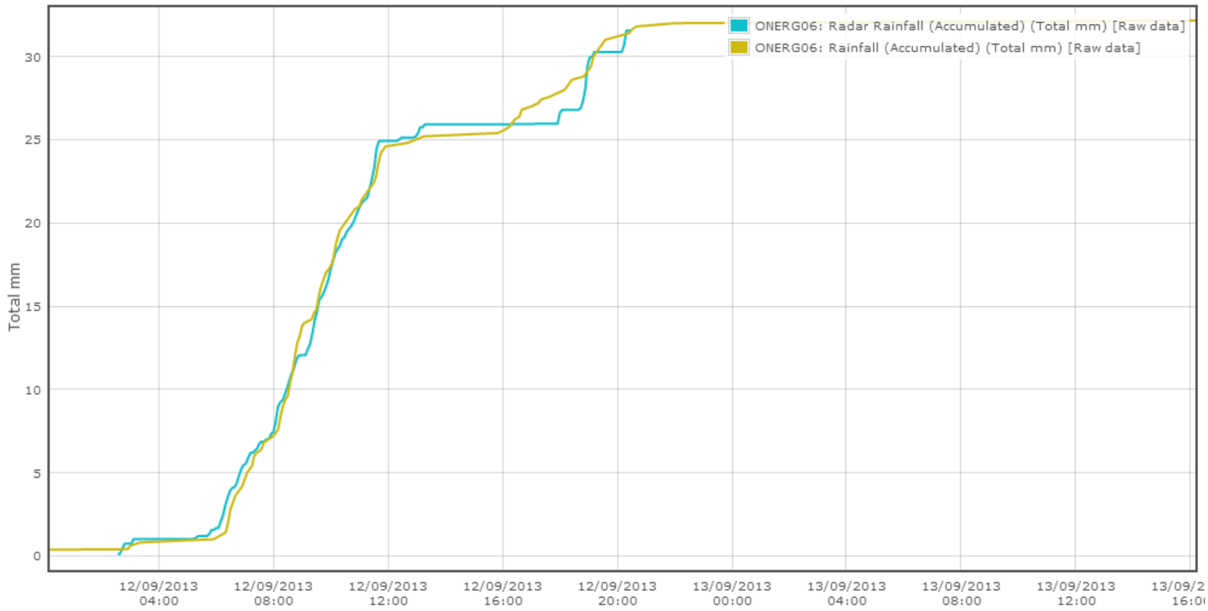

Figure 4: Radar Rainfall Calibration Results for Event 12/09/2013 - 12/09/2013

Table 1: Radar Calibration Results for Event 12/09/2013 - 12/09/2013

Rain Gauge	Radar Accumulation (mm)	Rainfall Accumulation (mm)	% Difference
ONERG01	16.92	19.2	12%
ONERG02	18.39	21.8	16%
ONERG03	21.99	28.6	23%
ONERG04	27.57	24.8	-11%
ONERG05	29.58	31	5%
ONERG06	31.56	31.8	1%
ONERG07	23.98	23.6	-2%
ONERG08	29.88	26	-15%

4.1.2 FLOW COMPARISON

The flow outputs from the model calibrated using rainfall gauge measurements and the comparison outputs from radar rainfall estimates were examined for a selected number of flow monitoring locations. The focus of the analysis was to determine if calibration parameters and subsequently catchment infiltration is being misrepresented using discrete point rainfall measurements. It was essential to compare only the two variations on the model to ensure consistency in the output and that the unknowns could be managed. It was also essential to ensure that the two comparative models produced an exact match during periods of no rainfall or outside of the radar data collection period. The model comparison primarily focused on single catchments with no upstream flow monitors and good of matches rainfall to the radar data match at the associated rainfall gauge to ensure that variance in the results was not a result of rainfall variation in the upstream catchments. Figures 5 through 7 below depict some of the major differences found between the two models for the 12/09 event. Table 2 summarises the results for flow monitor used for the model calibration.

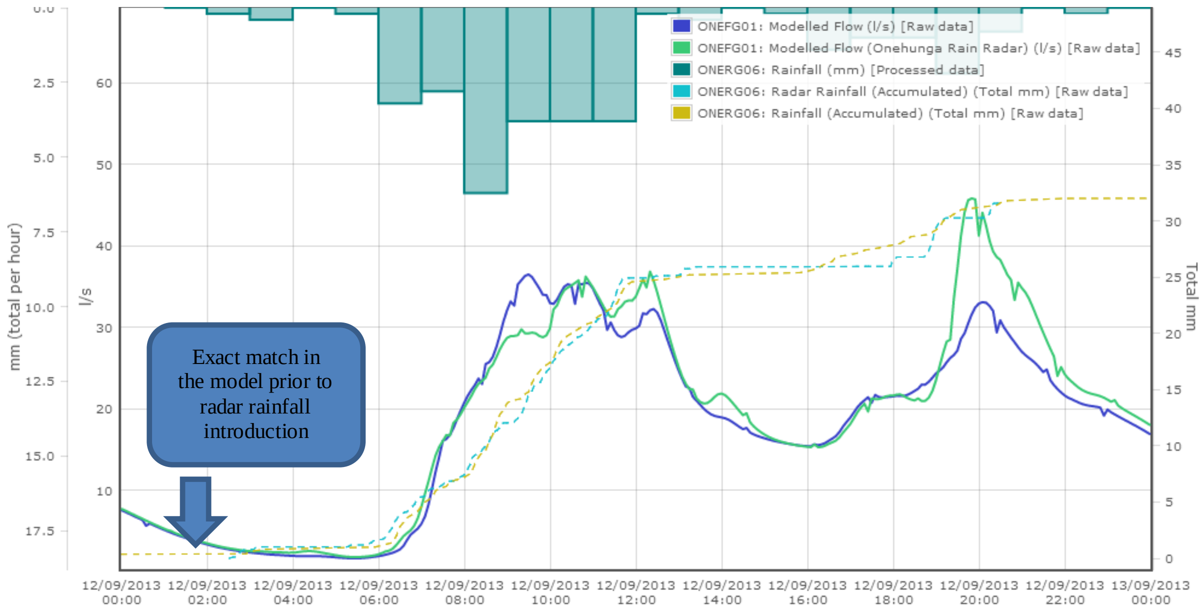

Figure 5: ONEFG01 Model Flow Rainfall Gauge vs Radar Rainfall for Event 12/09/2013 - 12/09/2013

ONEFG01 shows a reasonably good match during the majority of the event however shows a 24% difference in peak flow during the later stages. It is interesting to note that there is a relatively minor point wise increase in the radar intensity compared to the rain gauge at the rainfall gauges (see Figure 4), however it is not likely that this minor intensity spike would account for the magnitude of increase seen in the radar model. The discrepancy appears to be due to rain failing in the upper reaches of the catchment which are not adequately covered by the rain gauge network (Figure 6). At about 19:00 a convective rain band passes over the catchment. The heaviest rainfall is just to the north east of the study catchment, nonetheless the south western most extremity contributes accumulation to the catchment. Rain gauge ONER006 is less than 2km away from the heaviest rainfall, but receives only much lighter rain. Because the catchment is already saturated at the end the event and the extra intense rainfall in the top of the catchment readily infiltrates into the sewer network resulting in the higher model flow about 1 hour later.

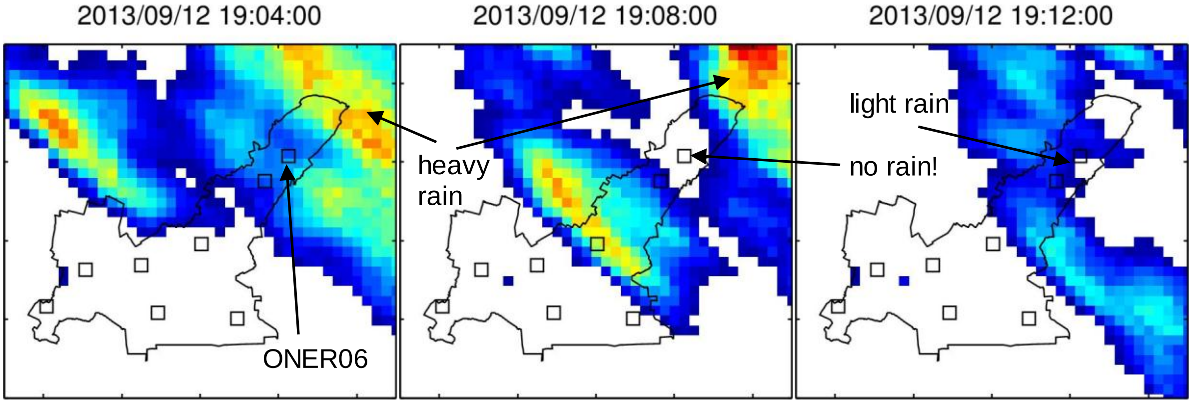

Figure 6: selected 2 minute radar rainfall intensity plots corresponding to Figure 5.

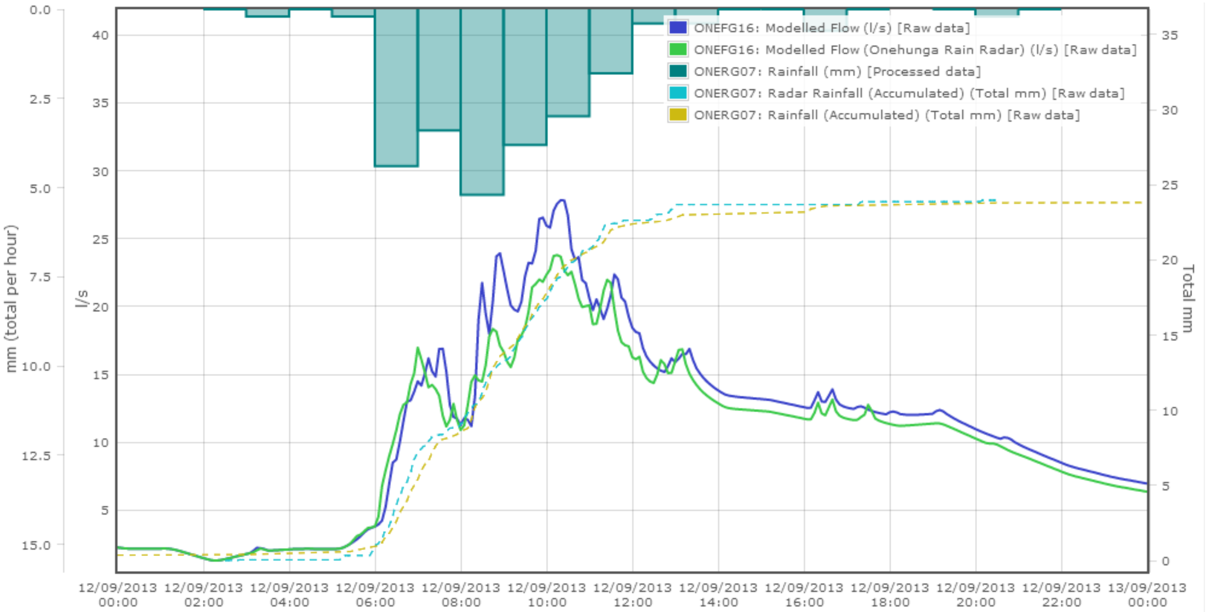

Figure 7: ONEFG16 Model FlowRainfall Gauge vs Radar Rainfall for Event 12/09/2013 - 12/09/2013

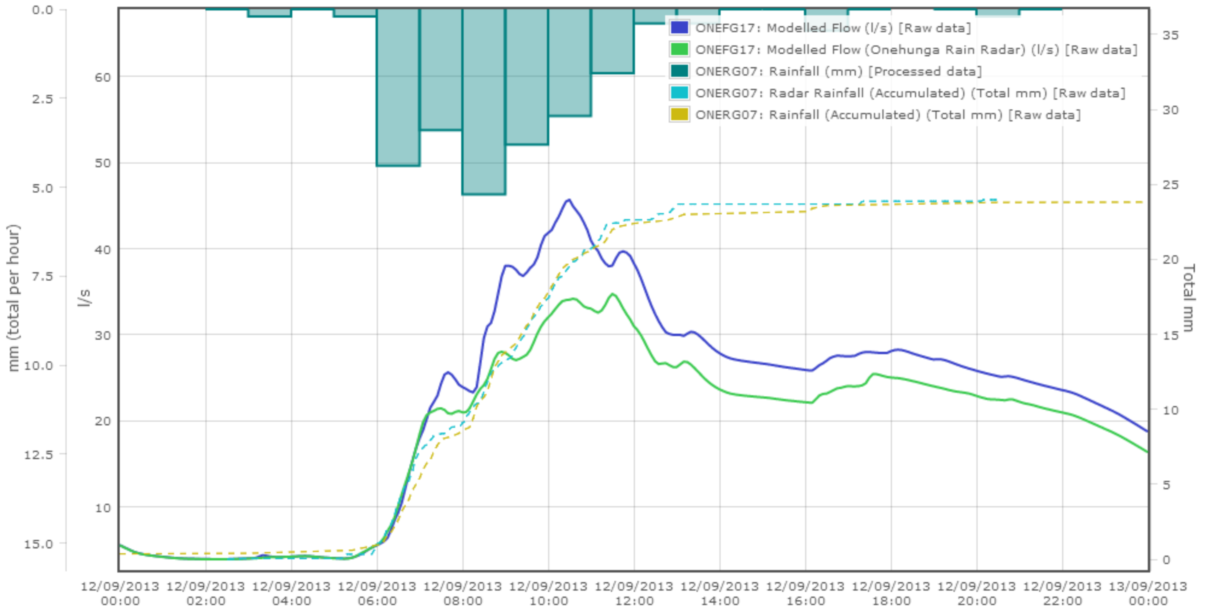

Figure 8: ONEFG17 Model Flow Rainfall Gauge vs Radar Rainfall for Event 12/09/2013 - 12/09/2013

ONEFG16 and ONEFG17 show a 14% and 24% difference respectively in peak flow during the peak of the event. The rain gauge and radar data is nearly identical at the rainfall gauge on a point wise basis. Once again, the radar rainfall distribution more correctly picked up spatial variability in the rain field (Figure 9). The radar indicates that the rainfall distribution did not completely cover the gauge sub-catchment, however the rain gauges have trouble resolving these gaps because their observations are propagated to the edges of the catchment where there are no observations, resulting in probable over-estimate of rainfall. . Over time, it is also likely that the rain-gauge driven model surface becomes wetter for the same reason, resulting in a further increase in inflow and infiltration later in the event. It was also noted that the flow at ONEFG17 is consistently overstated compared to the flow monitor at this location.

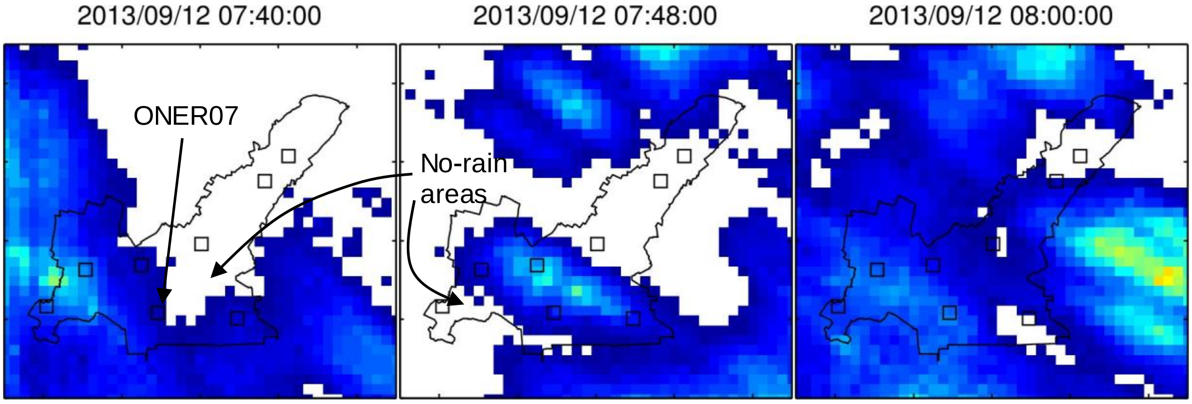

Figure 9: Selected 2 minute radar rainfall intensity plots corresponding to Figure 7 & 8.

Table 2: Model Flow Rainfall Gauge vs Radar Rainfall for Event 12/09/2013 - 12/09/2013

Flow Monitor Name	Radar Model Peak Flow (l/s)	Rain Model Peak Flow (l/s)	% Flow Diff	Correlation Coefficient	Radar Model Total Volume (m^3)	Rain Model Total Volume (m^3)	% Vol Diff	Rain Gauge	Radar Depth at RG (mm)	Rain Depth (mm)
ONEFG01	46	37	-24%	0.924	1636	1541	-6%	ONERG06	31.56	31.8
ONEFG02	16	17	6%	0.975	680	660	-3%	ONERG06	31.56	31.8
ONEFG03	69	68	-1%	0.941	3000	2860	-5%	ONERG05	29.58	31
ONEFG04	7	6	-17%	0.94	273	263	-4%	ONERG05	29.58	31
ONEFG05	52	49	-6%	0.972	2209	2170	-2%	ONERG05	29.58	31
ONEFG06	132	132	0%	0.976	6112	5902	-4%	ONERG06	31.56	31.8
ONEFG07	46	40	-15%	0.909	1889	1660	-14%	ONERG04	27.57	24.8
ONEFG08	11	10	-10%	0.955	358	336	-6%	ONERG04	27.57	24.8
ONEFG09	200	191	-5%	0.972	9045	8532	-6%	ONERG08	29.88	26
ONEFG10	17	17	0%	0.902	573	544	-5%	ONERG08	29.88	26
ONEFG11	18	17	-6%	0.968	540	528	-2%	ONERG04	27.57	24.8
ONEFG12	114	95	-20%	0.664	3290	2989	-10%	ONERG08	29.88	26
ONEFG13	51	54	6%	0.909	1339	1391	4%	ONERG03	21.99	28.6
ONEFG14	67	73	8%	0.963	2735	2684	-2%	ONERG07	23.98	23.6
ONEFG15	41	44	7%	0.924	1493	1539	3%	ONERG07	23.98	23.6
ONEFG16	24	28	14%	0.924	882	950	7%	ONERG07	23.98	23.6
ONEFG17	35	46	24%	0.811	1632	1895	14%	ONERG07	23.98	23.6
ONEFG18	25	25	0%	0.981	852	858	1%	ONERG02	18.39	21.8
ONEFG19	76	78	3%	0.954	2636	2699	2%	ONERG02	18.39	21.8
ONEFG20	94	106	11%	0.953	3910	4055	4%	ONERG02	18.39	21.8
ONEFG21	21	25	16%	0.95	1040	1109	6%	ONERG02	18.39	21.8
ONEFG22	438	462	5%	0.982	20323	19802	-3%	ONERG02	18.39	21.8
ONEFG23	33	37	11%	0.819	1165	1241	6%	ONERG01	16.93	19.2
ONEFG24	8	10	20%	0.931	222	222	0%	ONERG03	21.99	28.6
ONEFG25	7	8	13%	0.906	324	356	9%	ONERG02	18.39	21.8

It is interesting to highlight in Table 2 that the volume difference between the two models is relatively small in most instances. This suggests that the predicted excess rainfall is similar in both models. Indeed, the largest volume differences occurred in catchments for which there was a substantial discrepancy in the spatial distribution of rainfall according to the areal gauge estimate compared to radar (the examples discussed above). A larger variation which more closely related to the peak intensities across all of the catchments might have been expected. This difference can likely be attributed to the calibrated ground water infiltration model which stores excess rainfall in soil prior to a slower release into the system. It is possible that the averaging effects of the rain gauge measurements tend to result in more spread out estimates of rainfall and hence a bias towards the slow release model. On the other hand, the localised intensities detected by the radar may temporarily saturate sub-catchments resulting in fast release and higher peak flows even though the overall volumes are the same.

5 CONCLUSIONS

In general the rainfall radar model and rain gauge model compare relatively well. This indicates that high resolution radar data is capable of producing similar results to rainfall gauge measurements. The 8 rainfall gauges in the catchment appeared to provide ample coverage to define most of the events captured by the radar. This is supported by the tight correlation witnessed between the rain and radar model outputs shown in Table 2. However, as expected there are some catchments that showed considerable variation between the two models which can only be attributed to spatial-temporal rainfall variability.

For the September event analysed the volume difference between the two models remained minor, presumably buffered by the soil store in the ground water infiltration model. Peak intensities however do vary highlighting spatial variation in short localised rainfall bursts in the radar.

The case study highlights that high resolution radar data estimates can accurately predict rainfall across a catchment. It also supports that spatial-temporal variation highlighted within the radar data is not always well captured by discrete rainfall measurements and can have a significant impact on predicted peak flows and likely subsequent system performance and inflow and infiltration predictions. Although it is believed that the rainfall gauge density in this case study provided ample coverage it is not always possible to deploy rainfall gauges at this density especially in larger city wide catchment models.

In conclusion rainfall radar provides a significant amount of additional information that can be confidently used to further our understanding of the rainfall to runoff phenomenon that occurs in both wastewater and storm water catchments. It is in the opinion of the authors that the use of this type of data will become critical in future modelling projects, especially in larger areas with high rainfall variability like the Auckland isthmus. The true advantage is the added confidence in model calibration, ability to better understand and investigate different catchment model responses, and ultimately the large scale capital projects driven from model outputs.

6 LOOKING FORWARD

In this analysis we only examined one rainfall event in detail over the entire monitoring period. Several other events were captured and statistics were analysed in less detail with similar results witnessed. The analysis presented above highlights that there are some significant differences in the predicted peak flows between the rain and radar models. Ultimately what is most important question is will these differences skew our vision of the catchment performance and subsequent capital improvements (to manage peak flows) driven from the models. Additional analysis should be carried out to answer these questions.

The use of rainfall radar data should be given more consideration in larger and longer term catchment studies. International research has consistently shown that radar data can be confidently used for managing and controlling real-time sewer and storm water control systems where good time and space rainfall resolution is essential.

ACKNOWLEDGEMENTS

The Authors would like to thank Watercare Services Limited for allowing us to utilise the base monitoring data, calibrated model, and providing access to a site for the radar unit deployment. The University of Auckland's support in providing the high resolution radar hardware is also appreciated.

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