AN APPROACH TO THE CALCULATION OF PIPE FRICTION LOSS

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ABSTRACT

Pattle Delamore Partners Ltd. (PDP) has undertaken the detailed design for the irrigation component of Blenheim Sewage Treatment Plant upgrade. The scope of this work involved the design of K-Line and Dripline systems used to distribute treated effluent from the Sewage Treatment Plant.

This paper will discuss the method of calculating pipe losses based on the implicit solution. In the past, spreadsheet software, such as Microsoft Excel, has not been able to precisely calculate implicit formula due to its limitation in performing iterative calculations. Complex empirical equations have been approximated in the past to enable mass computation of the pipe frictional loss. Earlier version of Excel used explicit formula to compute an approximation of the implicit solution, however starting from its 2003 version, implicit formulas are now able to be calculated en masse.

The main advantage of using the implicit solution over the approximations for the pipe loss calculation is that unlike the approximations, there are no constraints on pipe sizes and flow rates in the implicit formula.

This paper will compare results obtained using the implicit solutions and the approximations and discuss the implications of not selecting the correct approximation for the pipe size and flows tested in the design.

KEYWORDS

Irrigation, Pressurised Water, Pipe Loss Calculation, Modelling

1 INTRODUCTION

Calculation of pipe friction loss has been used since the beginning of the 20th century to determine the amount of head loss incurred along the pipe length in order to design the pipe size, alignment, and the pump size in pressure mains. The pipe friction loss calculation generally uses the Darcy-Weisbach formula (1).

Darcy's friction loss formula contains a dimensionless pipe friction factor (λ) which could be determined using the Moody Diagram (Figure 2) or calculated using the Colebrook-White formula (3) for turbulent flows and the Darcy friction factor formula (2) for laminar flows. Due to the complexity of the Colebrook-White formula to calculate the pipe friction loss, several explicit approximations were used in the past. These approximations carry their own limitation, which will be outlined in detail in the following sections.

Following a description of the implicit and explicit methods, examples as to the accuracy of these methods are presented along with the suitability of these for design purposes.

2 PROJECT BACKGROUND

The Blenheim Sewage Treatment Plant (BSTP) resource consent authorising the discharge of treated wastewater from the plant to the Opawa River expired in 2008. In preparing an application to renew its consent, the Marlborough District Council (MDC) commissioned an investigation of options available for an alternative treated effluent disposal method, taking into account the needs to accommodate the predicted increases in both residential population and commercial development in the vicinity of the plant.

Subsequently, as a result of the investigations and consultation with the local iwi, MDC adopted a scheme which involved the decommissioning of the existing Opawa River outfall, irrigation of treated effluent onto the MDC owned land surrounding the BSTP, and discharge of effluent that is not able to be irrigated through constructed wetlands to a new outfall in the Wairau Estuary on the ebb tide. This scheme was granted resource consents in 2010.

The discharge consent to land included conditions which required that the discharge of wastewater to land shall be via drip or spray irrigation in the designated irrigation land area. The conditions specifically call for the use of a surface or subsurface drip irrigation in the areas located within 25 metres of the site boundary and public walking tracks, except for the western boundary adjoining the neighbouring land, where only surface or subsurface drip irrigation is to be used within 80 metres of the site boundary. For all other areas of the site, spray irrigation may be used.

PDP was engaged by MDC to design the K-Line spray irrigation and the drip irrigation component of the project.

The irrigation area was split into 7 drip irrigation and 14 spray irrigation zones (Figure 1). These were based on the existing fence lines and other physical constraints (such as surface drains that are unable to be altered).

For the preliminary design of the irrigation system Centre Pivot irrigators were proposed. However, to make these work within the constraints of the consent required significant modification to the surface drain that cross the site. Based on the engineering cost estimate, MDC decided to pursue a K-Line irrigation option over the Centre Pivot irrigation. PDP had not anticipated this decision and therefore had not budgeted on purchasing specific irrigation design software. As a result, it was decided to carry out the calculations of the friction loss from the first principles using Microsoft Excel. This paper describes the background to the methodologies available to calculate friction losses, how the calculations were carried out, and issues found using explicit vs implicit formula for calculating friction loss, along with an example of its use for the Blenheim STP project.

Figure 1: K-Line and Dripline Irrigation Areas

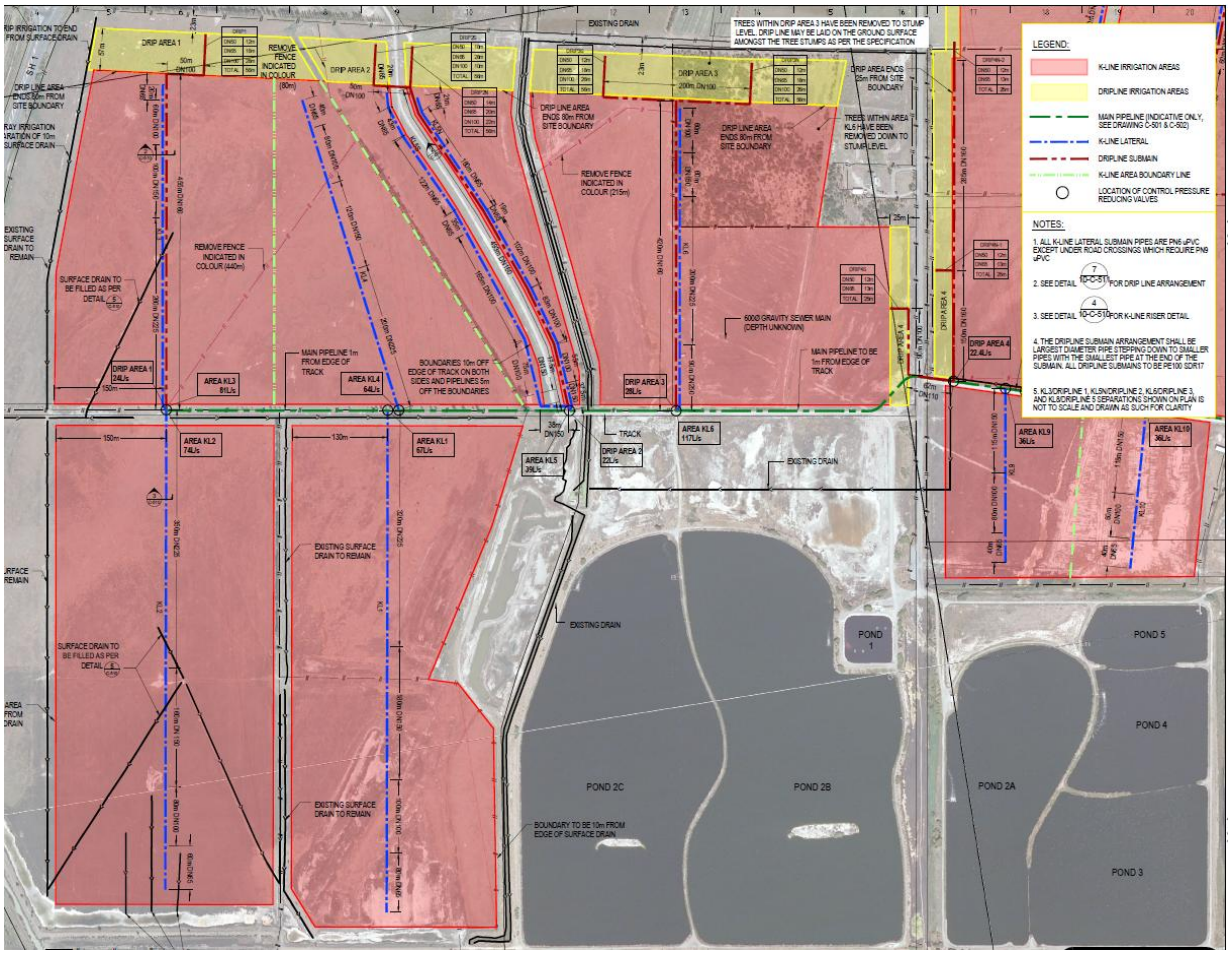


Figure 1 shows the areas covered by drip and spray irrigation. A total of 163 ha – 141.5 ha of spray irrigation area and 21.5 ha of drip line areas are irrigated.

3 PIPE LOSS CALCULATION

For the Blenheim STP project the friction loss calculation is essential particularly, as the K-Line flows are greatly influenced by the amount of water pressure supplied to the sprinklers/drip feeders. To achieve an even irrigation application rate over each irrigation zone, it was important to minimise the pressure variance between sprinklers/drip lines as the spacing between sprinklers are set at a constant interval.

The submain pipe sizes used in this project varied between DN 40 to DN 250, and the pipe material used is PVC-U.

Calculation of the head loss due to pipe friction is based on the Darcy-Weisbach formula (1).

$$h_f = \lambda \frac{l}{d} \frac{v^2}{2g} + \Sigma K \frac{v^2}{2g} \quad (1)$$

Friction Factor (λ) is the most influential factor to determine the pressure loss, which occurs within the pipe under a specific flow condition. λ can be calculated using the Darcy friction factor formula (2) for laminar flows or the Colebrook-White formula (3) turbulent flows. The Colebrook-White formula (3) could be calculated implicitly or using the explicit approximations, to be discussed further in the following sections.

$$\lambda = \frac{64}{\mathrm{Re}} \quad (2)$$

$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{k}{3.72d} + \frac{2.51}{\mathrm{Re}\sqrt{\lambda}}\right) \quad (3)$$

To calculate the Friction Factor (λ), Reynolds Number (Re) (4) needs to be determined.

$$\mathrm{Re} = \frac{Q}{A} \frac{D_H}{\nu} \quad (4)$$

For the circular pipes used in this project, $D_H = D$. The kinematic viscosity - ν of water (the type of liquid used in this project) is in the order of 1 x 10^{-6} m^2/s.

Calculation of the pipe friction factor (λ) using the Colebrook-White formula is limited to a Reynold Number (Re) of between 4 x 10^3 to 1 x 10^8 (transition and turbulent flows) and values of relative roughness of up to 0.05. A judgment call is required for flows with a Reynold Number or between 2,300 and 4,000, as it is the transition flow region where both the laminar and turbulent flow characteristics may appear.

Please note that in the following sections, k (the absolute measured pipe roughness) is interchangeable with ε (the absolute pipe roughness) to represent pipe roughness in the equations.

3.1 EXPLICIT APPROXIMATION

There are a number of explicit formulas used to approximate the pipe friction factor based on its flow characteristics as shown by its Reynolds Number (Re).

The explicit approximation used in this example is the Barr approximation (5), which was developed in 1981 (Marriott, 2009).

$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{k}{3.7d} + \frac{5.02\log\left(\frac{\mathrm{Re}}{4.518\log(\mathrm{Re}/7)}\right)}{\mathrm{Re}\left(\frac{1+\mathrm{Re}^{0.52}}{29(d/k)^{0.7}}\right)}\right) \quad (5)$$

The limitations to the Barr explicit approximation was not clearly stated in the original paper (Fang, et al., 2011).

The explicit formula has Friction Factor (λ) on one side of the equation, which could easily be solved using Microsoft Excel for the variable submain flow and pipe sizes.

This approximation was potentially helpful in this project, as there are thirty sub-main sections in a single irrigation area's sub-main, each with a variable flow and pipe size. However, it turns out that this formula is limited to the turbulent flow condition in a large pipe/open channel flow. In a small diameter pipe flow, this approximation is unable to yield an accurate result.

3.1.1 EXPLICIT APPROXIMATION VERIFICATION

A problem with the explicit formula was discovered through a simple check where the pipe's roughness (k) value was changed from 0.007 mm for PVC pipes to 0.006 mm to allow for some slime build up in the pipes.

In theory, pipes with higher roughness coefficient should have higher λ. However, in this case the approximation decreased the λ when k was increased.

As an example, for water flow of 62 l/s in a 25 m long, 225 mm PVC pipe resulted in a Re of 3.49×10^5. With a k value of .007 mm, Barr's approximation estimates a λ of 0.0406 (which equates to a head loss of 0.55 m), while increasing the k value to 0.6 mm, Barr's approximation decreased the value of λ to 0.0267 (which equates to a head loss of 0.36 m). This verification results in questions over the appropriateness of this approximation for the pipes used in the project, as the approximation did not pass the logic check. Therefore, another equation/approximation is needed to correspond with the Moody Diagram in an automated manner.

3.2 IMPLICIT FORMULA

The implicit formula is based on Colebrook-White equation (3), which has Friction Factors (λ) on both sides of the equation that requires an iterative method to solve for the Friction Factor.

In the past, the solver function in Microsoft Excel was used to determine the friction factor, however the solver was heavily constrained as it could only solve one factor at a time for a specific flow and pipe size. In a project where there is significant variation in flow and pipe size, the use of solver is not only time consuming but it is also impractical.

The latest version of Excel allows the iterative solution where a circular cell reference occurs. This enables the calculation of friction factors en masse for variable flow and pipe size as a condition. It has opened a new opportunity for designers to obtain an exact friction factor value from the given flow and pipe size condition without any limitation on its flow characteristics shown by the Reynolds Number.

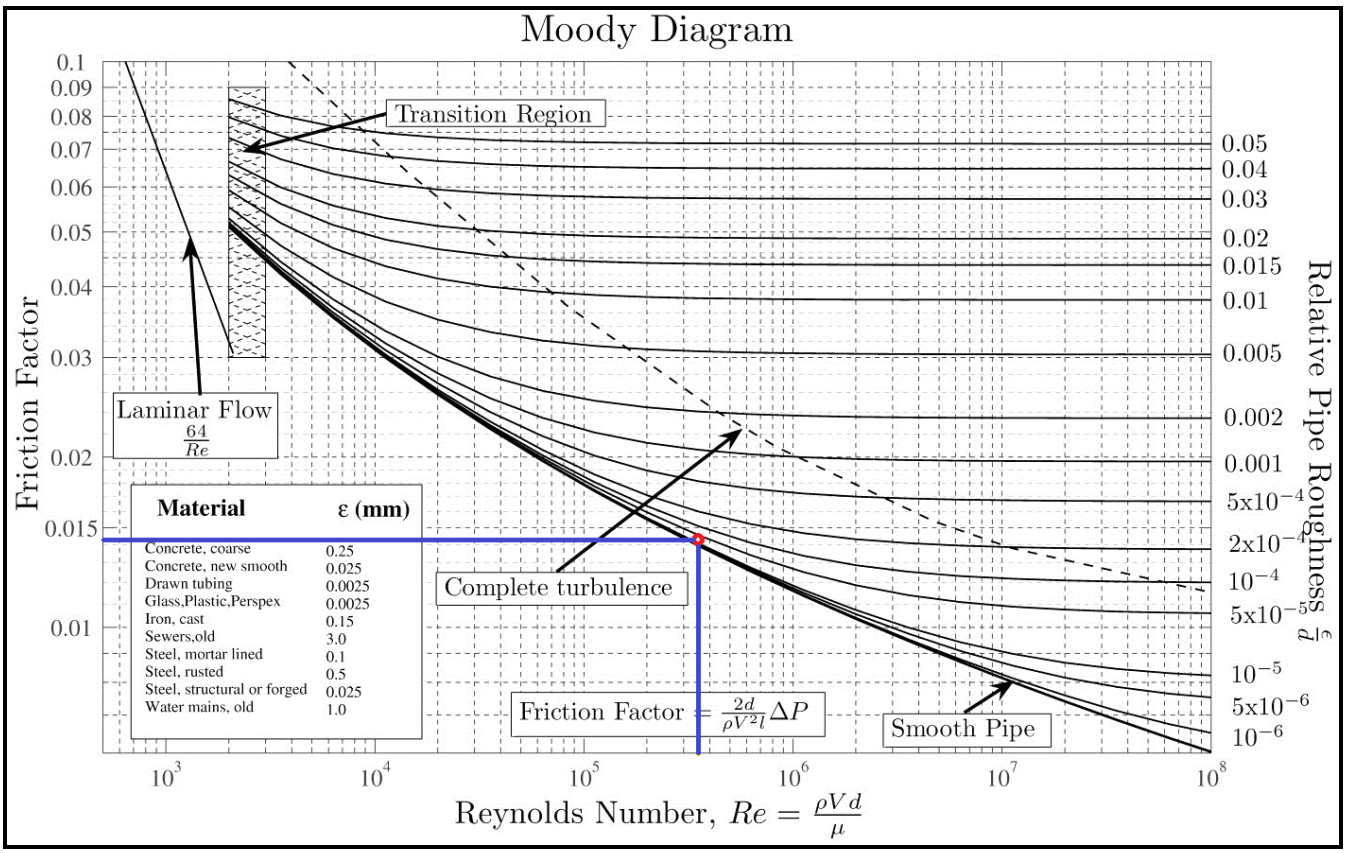
3.2.1 IMPLICIT FORMULA VERIFICATION

To verify that issues associated with the explicit formula are not going to affect the implicit formula, the same test is applied to the formula, which resulted in a λ of 0.0144 (which equates to a head loss of 0.20 m) for a k value of 0.007 mm and a λ of 0.0257 (which equates to a head loss of 0.35 m) for a k value of 0.6 mm. The increase in λ due to the increase in roughness coefficient is consistent with the logic behind the formula.

4 FORMULA VERIFICATION

Both formulas were tested against the Moody Diagram (Moody, 1944) (Figure 2). As an example, for water flow in a 25 m long, 225 mm PVC pipe (roughness coefficient ε of 0.007 mm) resulted in a Re of 3.49 ×10^5. Barr's approximation estimates a λ of 0.0406. Using Colebrook's implicit formula, the λ is in the order of 0.0144.

Figure 2: Moody Diagram



When compared on the Moody Diagram, with a relative pipe roughness $\frac{\varepsilon}{d}$ value of 3.11×10^{-5}, the friction factor shown in the diagram is in the order of 0.0140. This verification shows that the iterative Colebrook White formula calculation is in agreement with the Moody Diagram, and the Barr approximation method yielded an inaccurate factor in this instance.

5 FORMULA COMPARISON

Further research on the Barr approximation yielded an alternative Barr equation (6) (Barr, 1981).

$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{k}{3.7d} + \frac{4.518\log\frac{\mathrm{Re}}{7}}{\mathrm{Re}\left(1+\frac{\mathrm{Re}^{0.52}}{29}\left(\frac{k}{d}\right)^{0.7}\right)}\right) \quad (6)$$

Table 1 shows the comparison between Barr approximations using equation (5) and equation (6) and the Implicit Colebrook-White Formula to determine the limitation of the approximation. The comparisons are based on the following constant variables:

- Water kinematic viscosity of 1 x 10^{-6} m^2/s
- Pipe length of 100 m

Table 1: Comparison of Pipe Friction Loss Calculation

Pipe Diameter (mm)	**Pipe Flow (l/s)**	**Pipe Velocity (m/s)**	**Reynolds Number (Re) (x 10^5)**	**Pipe Friction Loss (m/100m)**					
				k = 0.6 mm			k = 0.007 mm		
				Barr (5)	Barr (6)	Colebrook-White (3)	Barr (5)	Barr (6)	Colebrook-White (3)
50	2	1.02	0.51	4.62	4.39	4.37	8.60	2.26	0.25
75	5	1.13	0.85	3.27	3.12	3.13	5.73	1.65	1.66
100	10	1.27	1.27	2.81	2.69	2.70	4.64	1.44	1.45
150	20	1.13	1.70	1.33	1.26	1.26	2.32	0.71	0.72
200	40	1.27	2.55	1.15	1.10	1.10	1.90	0.63	0.63
250	60	1.22	3.06	0.80	0.76	0.76	1.35	0.45	0.45
300	90	1.27	3.82	0.69	0.65	0.66	1.14	0.39	0.39
375	140	1.27	4.75	0.52	0.49	0.49	0.86	0.30	0.30
450	200	1.26	5.66	0.41	0.39	0.38	0.68	0.24	0.23
500	250	1.27	6.37	0.36	0.34	0.35	0.60	0.21	0.21
575	330	1.27	7.31	0.31	0.29	0.29	0.51	0.18	0.18
750	550	1.24	9.34	0.21	0.20	0.20	0.35	0.13	0.13
1050	1,100	1.27	13.34	0.15	0.14	0.14	0.24	0.09	0.09

Table 1 shows that Barr Approximation (5) is within 7% of head loss when compared to the Implicit Formula calculated loss when the pipe has a roughness coefficient (k) value of 0.6 mm (e.g. for concrete pipes conveying a mixture of stormwater and sewer). While Barr Approximation (6) is within 3% of head loss calculated by the Implicit Formula using the same k value.

Table 2: Accuracy of the Approximations for a Variety of Pipe Sizes

Pipe Diameter (mm)	k = 0.6 mm		k = 0.007 mm	
	Barr (5)	Barr (6)	Barr (5)	Barr (6)
50	5.72%	0.46%	3340.00%	804.00%
75	4.47%	-0.32%	245.18%	-0.60%
100	4.07%	-0.37%	220.00%	-0.69%
150	5.56%	0.00%	222.22%	-1.39%
200	4.55%	0.00%	201.59%	0.00%
250	5.26%	0.00%	200.00%	0.00%
300	4.55%	-1.52%	192.31%	0.00%
375	6.12%	0.00%	186.67%	0.00%
450	7.89%	2.63%	195.65%	4.35%
500	2.86%	-2.86%	185.71%	0.00%
575	6.90%	0.00%	183.33%	0.00%
750	5.00%	0.00%	169.23%	0.00%
1050	7.14%	0.00%	166.67%	0.00%

Table 2 shows that Barr Approximation (5) is significantly different (over 150% off) when used to calculate the head loss when the pipe has a lower roughness coefficient – such as when conveying fresh water though a plastic pipe, the pipe losses increased as the roughness coefficient decreases (contrary to the logic and the calculated loss using the implicit formula). While Barr Approximation (6) is within 4% of head loss calculated by the Implicit Formula using the same k value for pipes that have a diameter of 75 mm or more.

Therefore it could be concluded that both of the Barr approximations are only suitable for rough pipes (with a k value of 0.6 mm and above) for a relatively accurate (within 7% of the true value) approximation. While Barr approximation (6) is relatively more accurate than Barr approximation (5) for smooth pipes (k of 0.007 mm in this instance), it is still inaccurate for pipes with diameters of 50 mm or less.

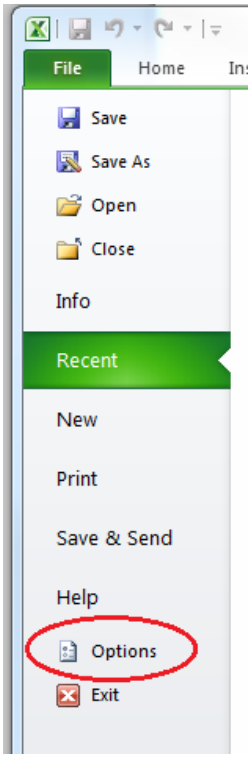
Note that there are other approximations which are suitable for a variety of pipe conditions such as pipe size, velocities and pipe roughness. As an example, Table 3 shows a variety of explicit approximations and the constraints associated with each equation (Fang, et al., 2011).

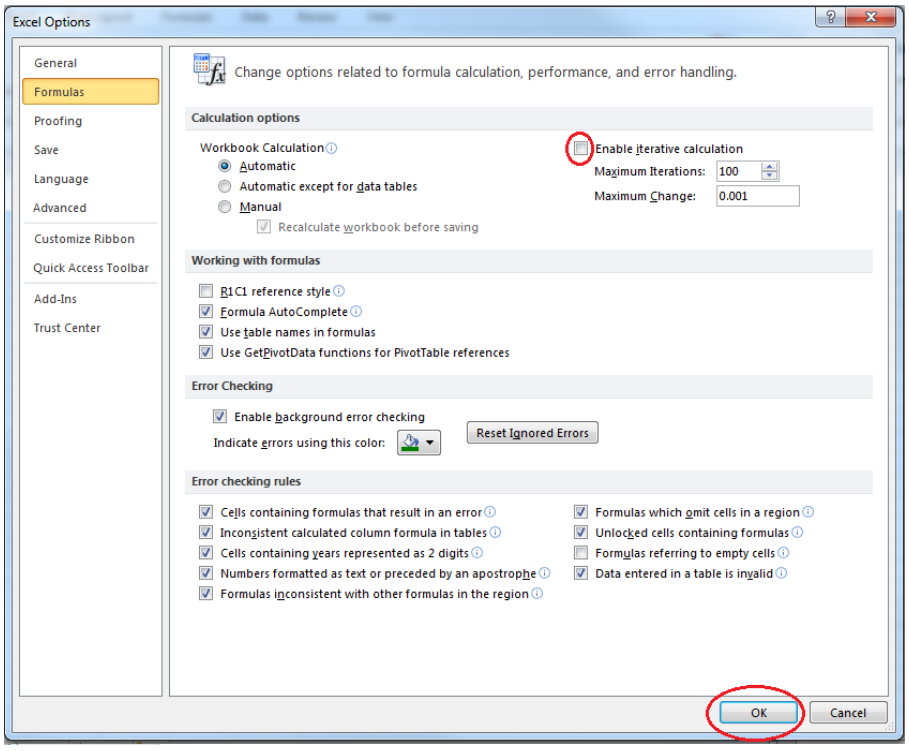
Table 3: Comparison of Explicit Approximations and Its Limitations

Explicit Approximation	**Formula**	**Limitation**
Blasius (1913) (Fox, et al., 2010)	$\lambda = 0.079\mathrm{Re}^{-\frac{1}{4}}$	$3000 < \mathrm{Re} < 10^5$ Only valid for smooth pipe
Haaland (Haaland, 1983)	$\frac{1}{\sqrt{\lambda}} = -1.8\log\left[\left(\frac{k}{3.7d}\right)^{1.11} + \frac{6.9}{\mathrm{Re}}\right]$	$10^{-6} \le \frac{k}{d} \le 0.05$ $4000 \le \mathrm{Re} \le 10^8$
Jain and Swamee (Swamee & Jain, 1976)	$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{k}{3.7d} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)$	$10^{-6} \le \frac{k}{d} \le 0.05$ $5000 \le \mathrm{Re} \le 10^8$

6 IMPLEMENTATION

The iterative calculation function is not enabled in Microsoft Excel by default. To enable it, requires carrying out the following steps:





Once enabled, the Colebrook-White's equation (3) could be inserted into the cell, which refers to itself in the iterative calculation to determine the friction factor.

E2 f_x =IF(D2="","",-2*LOG((2.51*E2/D2+L$2/A2/3.72)))

	A	B	C	D	E	F	G	H
1	Pipe Dia (mm)	Length (m)	Flow (L/s)	Re #	1/sqrt(λ)	Friction Factor (λ)	Head Loss	Velocity (m/s)
2	238	20	59	3.14E+05	8.26E+00	0.014650518	0.11	1.32053

An example on how this was used for the BSTP Irrigation work, based on the K-Line Area 1 is shown in (Figure 3).

Figure 3: BSTP K-Line Area 1 Pipe Loss Calculation

Name	KL1		$h_f = -\lambda\frac{l}{d}\frac{v^2}{2g} + \Sigma K\frac{v^2}{2g}$					
Length of Pipe	695							
Roughness of pipe	0.007 mm							
Lateral spacing	20 m		Offset to First Lateral	20 m				
Flow at start	61.7 L/s							
Flow at end	1.8 L/s							
Head at start	40 m							
Kinematic visocisty	1.00E-06							
Head Loss	6.18 m							

$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{k}{3.72d} + \frac{2.51}{\mathrm{Re}\sqrt{\lambda}}\right)$$

Submain Diameter	Chainage (m)	Head	Flow	Re #	1/sqrt(λ)	Friction Factor (λ)	Head Loss	Velocity (m/s)
237.7	0	40	61.7	3.30E+05	8.30E+00	0.0145	0.120	1.39
237.7	20	39.88	59.9	3.21E+05	8.28E+00	0.0146	0.114	1.35
237.7	40	39.77	58.0	3.11E+05	8.26E+00	0.0147	0.108	1.31
237.7	60	39.66	56.2	3.01E+05	8.23E+00	0.0148	0.102	1.27
237.7	80	39.56	54.4	2.91E+05	8.21E+00	0.0148	0.096	1.23
237.7	100	39.46	52.6	2.82E+05	8.18E+00	0.0149	0.090	1.19
237.7	120	39.37	50.8	2.72E+05	8.16E+00	0.0150	0.084	1.14
237.7	140	39.29	49.0	2.62E+05	8.13E+00	0.0151	0.079	1.10
237.7	160	39.21	47.2	2.53E+05	8.11E+00	0.0152	0.074	1.06
237.7	180	39.13	45.3	2.43E+05	8.08E+00	0.0153	0.069	1.02
237.7	200	39.07	43.5	2.33E+05	8.05E+00	0.0154	0.064	0.98
237.7	220	39.00	41.7	2.23E+05	8.02E+00	0.0156	0.059	0.94
237.7	240	38.94	39.9	2.14E+05	7.99E+00	0.0157	0.054	0.90
237.7	260	38.89	38.1	2.04E+05	7.95E+00	0.0158	0.050	0.86
237.7	280	38.84	36.3	1.94E+05	7.92E+00	0.0160	0.046	0.82
237.7	300	38.79	34.5	1.85E+05	7.88E+00	0.0161	0.042	0.78
151.2	320	38.75	32.7	2.75E+05	8.12E+00	0.0151	0.338	1.82
151.2	340	38.41	30.8	2.60E+05	8.09E+00	0.0153	0.304	1.72
151.2	360	38.11	29.0	2.44E+05	8.04E+00	0.0155	0.272	1.62
151.2	380	37.84	27.2	2.29E+05	8.00E+00	0.0156	0.242	1.52
151.2	400	37.59	25.4	2.14E+05	7.95E+00	0.0158	0.213	1.41
151.2	420	37.38	23.6	1.99E+05	7.90E+00	0.0160	0.186	1.31
151.2	440	37.20	21.8	1.83E+05	7.84E+00	0.0163	0.161	1.21
151.2	460	37.03	20.0	1.68E+05	7.78E+00	0.0165	0.138	1.11
151.2	480	36.90	18.1	1.53E+05	7.71E+00	0.0168	0.116	1.01
107.7	500	36.78	16.3	1.93E+05	7.85E+00	0.0162	0.494	1.79
107.7	520	36.29	14.5	1.72E+05	7.76E+00	0.0166	0.398	1.59
107.7	540	35.89	12.7	1.50E+05	7.67E+00	0.0170	0.312	1.39
107.7	560	35.58	10.9	1.29E+05	7.56E+00	0.0175	0.236	1.19
107.7	580	35.34	9.1	1.07E+05	7.44E+00	0.0181	0.170	1.00
71	600	35.17	7.3	1.30E+05	7.53E+00	0.0176	0.850	1.83
71	620	34.32	5.4	9.76E+04	7.33E+00	0.0186	0.504	1.37
71	640	33.82	3.6	6.51E+04	7.05E+00	0.0201	0.243	0.92
71	660	33.57	1.8	3.25E+04	6.55E+00	0.0233	0.070	0.46
	680	33.50						

Pipe Dia	Length (m)	Submain Riser
65 mm	80	4
100 mm	100	5
150 mm	180	9
225 mm	320	16
250 mm	0	0
	680	**34**
Avg Head	37.6 m	
No. of laters	34	
Min Head	33.5 m	

In order to maintain a velocity range of between 0.4 m/s and 1.5 m/s as specified for the Blenheim irrigation project, a variety of submain sizes are used to convey a variety of flows, divided into 20 m sections (the distance between risers in the submain). In this setup, the submain pipe sectional loss could be easily calculated for a variety of pipe sizes in order to minimise head losses, while operating within the pipe velocity constraints.

7 CONCLUSION

From the example above, it is clear that the implicit formula has a distinct advantage over the explicit approximation which is traditionally used to simplify the friction factor calculation. While there are a number of explicit approximations, they all have limitations as to the range of conditions over which they are suitable. This project highlights the importance of understanding those limitations before these approximations are used. However, with the improvement in the readily available spreadsheet software such as Microsoft Excel, it is now able to perform iterative calculations for variable flow and pipe size condition, to obtain a much more accurate answer without risking errors from the inappropriate use of the wrong equations.

SYMBOLS USED IN THE FORMULA

The symbols used in the formula are described in Table 4.

Table 4: Symbols Used in the Formula and Its Units

Symbol	Description	Unit
Q	Flow Through The Pipe	m^3/s
λ	Friction Factor	-
g	Gravitational Force	m^2/s^2
h_f	Head Loss	m
ν	Kinematic Viscosity	m^2/s
l	Length of Pipe	m
A	Pipe Cross Sectional Area	m^2
d or D or D_H	Pipe Inside Diameter	mm or m
k/d	Relative Roughness	-
k or ε	Roughness Coefficient (absolute)	mm or m
v	Velocity	m/s

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